Available online at www.sciencedirect.com
SCIENCE DIRECT®
International Journal of Thermal Sciences 44 (2005) 1123–1130
International Journal of Thermal Sciences
www.elsevier.com/locate/ijts

Svelteness, freedom to morph, and constructal multi-scale flow structures

S. Lorente [a], A. Bejan [b,*]

[a] *Laboratoire Matériaux et Durabilité des Constructions, Institut National des Sciences Appliquées, Département de Génie Civil, 135 Avenue de Rangueil, 31077, Toulouse, France*
[b] *Duke University, Department of Mechanical Engineering and Materials Science, Durham, NC 27708-0300, USA*

Received 25 April 2005; accepted 23 August 2005
Available online 26 October 2005

Abstract

This paper reviews recent progress on constructal theory and design. The emphasis is on the development of multi-scale, nonuniformly distributed flow structures that offer increased compactness (e.g., heat transfer density). Examples are counterflow heat exchangers with tree-shaped hot and cold streams, and tree architectures on a disc. Every flow system has a property called *svelteness* (*Sv*), which is the ratio between its external (global) length scale and its internal length scale ($V^{1/3}$), where V is the volume occupied by all the ducts. Emphasis is placed on the development of simple strategies for decreasing the computational cost required by the development of such structures. The generation of multi-scale flow configurations is a process that can be projected on a diagram having global performance on the abscissa and degrees of freedom on the ordinate. This process rules the development (evolution) of all flow configurations for systems with global objective, global constraints and freedom to morph.

1. Flow systems develop configuration

Constructal theory is a purely mental viewing of how the configuration of flow systems is generated everywhere, in nature and engineering. The generation of macroscopic flow configuration (shape, structure, architecture) is reasoned on the basis of a physics principle of maximization of flow access—the constructal law [1,2]:

> "For a finite-size flow system to persist in time, it must evolve in such a way that it provides easier access to the currents that flow through it".

The constructal law is about the natural tendency to maximize flow access. It is a generalization of the tendency of all things to flow along paths of least resistance, and of Sadi Carnot's vision to avoid friction and shocks, Darwin's survival of the fittest, Fermat's law of refraction, etc.

Constructal theory places on scientific grounds the process of design—the generation of configuration. By generating shape and structure from principle, constructal theory grasps the way nature approaches maximal flow efficiency (e.g., trees, respiratory systems, dendritic structures) and uses this knowledge to build the best performance in new engineering devices. The optimization of global system performance subject to constraints is identified as principle for the generation of geometric form (shape and structure) in systems with internal flows, engineered and natural. The geometric structures derived from this principle for engineering applications have been named constructal designs. The thought that the same principle serves as basis for the occurrence of geometric form in natural flow systems is constructal theory.

Resistances cannot be minimized individually and indiscriminately, because of constraints: space is limited, streams must connect components, and components must fit inside the greater system. Resistances compete against each other. The route to improvements in global performance is by balancing the reductions in the competing resistances. This amounts to spreading the imperfection through the system in an optimal way, so that the total imperfection is reduced. Optimal spreading is achieved by properly sizing, shaping and positioning the components. Optimal spreading means geometry and ge-

* Corresponding author.
E-mail address: abejan@duke.edu (A. Bejan).

1290-0729/$ – see front matter
doi:10.1016/j.ijthermalsci.2005.08.011

Nomenclature

d	distance between ports on the rim	m
D_i	tube diameter	m
f	dimensionless global flow resistance, Eq. (2)	
L	disc radius	m
L_i	tube length	m
$\dot{m}$	mass flow rate	$kg \cdot s^{-1}$
n	number of pairing levels	
n_0	number of tubes that touch the center of the disc	
N	number of ports on the rim	
p	number of pairing levels	
R	flow resistance, $\dot{m}/\Delta P$	
$\widetilde{R}_t$	dimensionless global thermal resistance	
Re_D	tube Reynolds number	
Sv	svelteness number, Eq. (1)	
V	volume of all the ducts	m^3
$\widetilde{W}$	dimensionless pumping power	

Greek symbols

ΔP	overall pressure difference	Pa
ν	kinematic viscosity	$m^2 \cdot s^{-1}$

ography. In the end, the geometric system—its architecture—emerges as a result of global, integrative optimization.

There is a time arrow in how flow configurations evolve, and it points toward better flowing configurations. This time arrow is distinct from the time arrow of the second law (i.e., an isolated system evolves toward equilibrium). The constructal law is the law of geometry generation, while the second law is the law of entropy generation.

The progress on constructal theory and design is reviewed in Refs. [1–8], which show that the constructal law has been invoked to predict the occurrence of a variety of natural flow features such as optimal spacings, ducts with round cross-section, and tree-shaped networks for distribution and collection. In this paper we draw attention to several, more recent *design* applications, in which multi-scale hierarchical flow structures are used in order to increase the compactness (heat transfer density) of heat transfer devices. The focus is on structures with multiple length scales that are distributed nonuniformly, the rapid (low-cost) generation of such structures, and the role played by design freedom (the freedom to morph the configuration) in the maximization of global flow-system performance.

2. Flowing with svelteness

The constructal law is the law of the generation of flow geometry in general. It is not restricted to the generation of tree-shaped flows, although our group has devoted considerable effort to developing strategies for optimizing tree-shaped configurations more effectively and more accurately [9–17]. Fig. 1 is a fluid flow example [11] that may be regarded as a map of the world of tree-shaped flow designs. The tree-shaped flows summarized in Fig. 1 are simpler than most, because they connect a circle with its center. Liquid flows in Poiseuille regime through round tubes. The total tube volume (V) and the disc radius (L) are fixed. These are important characteristics of all flow systems.

In this paper, we identify a new property of flow systems. The size of a flow system can be measured in at least two ways [5]. One is the macroscopic dimension (L), which indicates the external size of the flow system. The other is internal size (V), or the internal length scale $V^{1/3}$. Because of these two dimensions, a flow system possesses *svelteness*, which is a new property defined as the ratio

$$Sv = \frac{\text{external length scale}}{\text{internal length scale}} = \frac{L}{V^{1/3}} \tag{1}$$

Svelteness is important for several reasons. Because space comes at a premium in design, the available space must be allocated mostly to the components that work, not to the flow distribution and collection network that serves the components that work. This calls for designs with large Sv values.

Svelteness is assumed, but never mentioned in the development of flow networks such as Fig. 1: the design assumption of Poiseuille flow in every tube implies a large Sv value. For example, if the tube length scales as L, and the tube diameter D as $V^{1/3}$, then the Poiseuille flow occupies most of the tube length when the tube entrance length $D\,Re_D$ is much shorter than L. This translates into the requirement $L/D \gg Re_D$, which means that $Sv \gg Re_D$. This is important, because only in the high-Sv range the local pressure losses (e.g., entrance, junctions) can be neglected. Only in this range the optimization of fluid trees is simple, as in Fig. 1 and most of the physiology and river-basin physics literatures.

3. Freedom vs. performance

Assume that in Fig. 1 the number of outlets (N) distributed equidistantly on the circle is fixed: this is equivalent to fixing the smallest scale, which is the distance between two neighboring ports, $d = 2\pi L/N$. The dimensionless number plotted on the ordinate (f) depends solely on flow configuration, and is a measure of the global flow resistance ($\Delta P/\dot{m}$):

$$f = \frac{\Delta P}{\dot{m}} \frac{V^2}{8\pi \nu L^3} = \frac{\Delta P}{\dot{m}} \frac{L^3}{8\pi \nu Sv^6}$$

One solid curve in Fig. 1 represents the lowest flow resistance that can be achieved by optimizing an architecture with a specified number (p) of levels of pairing or bifurcation. The design pictured in the inset is the optimized structure for $p = 2$, where pairing (dichotomy) is an optimization result, not an assumption (see Refs. [1,11]). All the non-optimal $p = 2$ designs fall above the $p = 2$ curve. The design space is divided into two regions by the rough envelope of the constant-p curves. Above the envelope reside the possible designs.

There is an infinity of possible configurations with the same L, V and N (or d). According to the constructal law, these configurations evolve in time toward lower f values. They

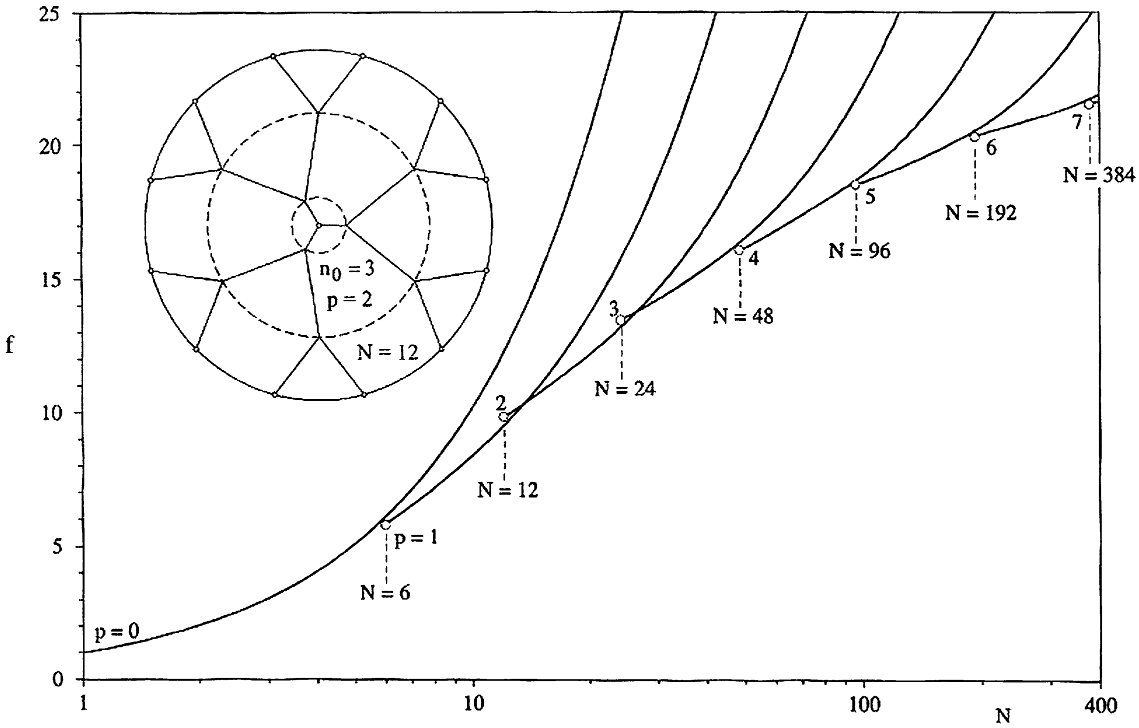

Fig. 1. The minimal flow resistance of networks of round tubes with Poiseuille flow [11].

migrate downward in the vertical cut made at $N =$ constant in Fig. 1. For example, when $N = 48$ the ultimate in this sequence of increasing freedom and minimizing f by morphing the structure is a structure with four levels of pairing. The moderate complexity represented by $p = 4$ is a result—an optimized architectural feature—not the object of a process of "maximization of complexity".

Children can draw patterns that are considerably more complex than the optimized tree shown in the inset of Fig. 1. This observation is important. Because the constructal architecture is deduced from principle, its complexity is deduced. Complexity, high or low, is an optimization result. It is not an objective. Optimized complexity must not be confused with maximized complexity.

The constructal law acts in $N =$ constant plane, which is shown schematically in Fig. 2. The world of possible designs occupies a portion of the domain in freedom-performance coordinates. According to the constructal law, the non-optimal configurations migrate to the left as time passes. Because they change and migrate, they are called *non-equilibrium flow structures* [5]. The progress toward higher performance (lower f) is aided by the increase in the degrees of freedom of the morphing structure. The ordinate of Fig. 2 shows the number of central tubes (n_0) that correspond to the optimized structures that emerge when the number of pairing levels (p) is specified. The best of all the optimized structures is the one with $p = 4$: this is the *equilibrium flow structure* [5]. Here the performance level has reached its maximum, the flow configuration enjoys maximum freedom to morph, and, if the configuration changes, its performance does not change.

The points indicated with $p =$ constant in Fig. 2 are singularities: they represent "peaks" of performance. They divide in a conceptual (impossible to draw) way the freedom-performance plane into two regions. To the right of the divide is the world of possible configurations. To the left of the divide is the inaccessible design domain. In time, the tendency of the possible flow configurations is to migrate toward the divide, toward greater performance.

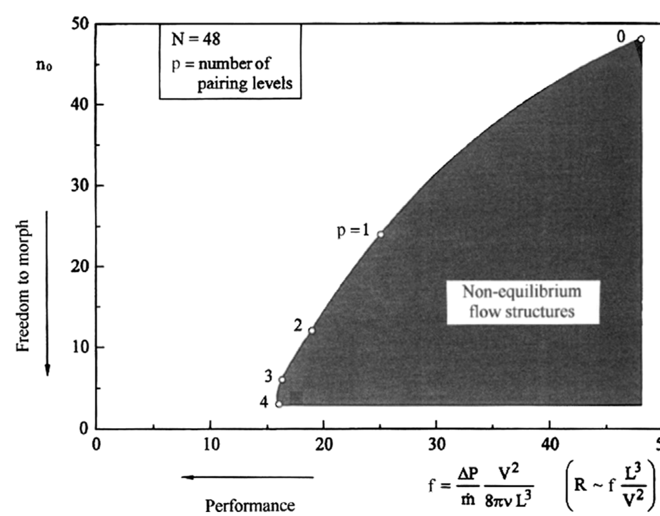

Fig. 2. The freedom versus performance world of possible flow configurations [5].

Fig. 1 has many analogs, in river morphology, cooled electronics, city traffic, lung design, etc. When the figure is swept from left to right, the animal species (the flow system) changes because the constraints change: the smallest length scale (d) decreases. In response, optimized complexity increases, and the corresponding (the minimized) flow resistance increases. Such

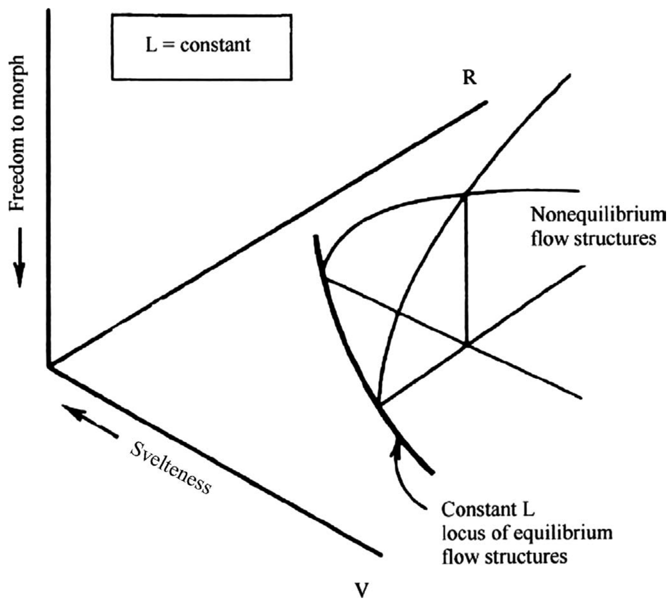

Fig. 3. The space inhabited by all the flow structures when the global external size (L) is fixed [5].

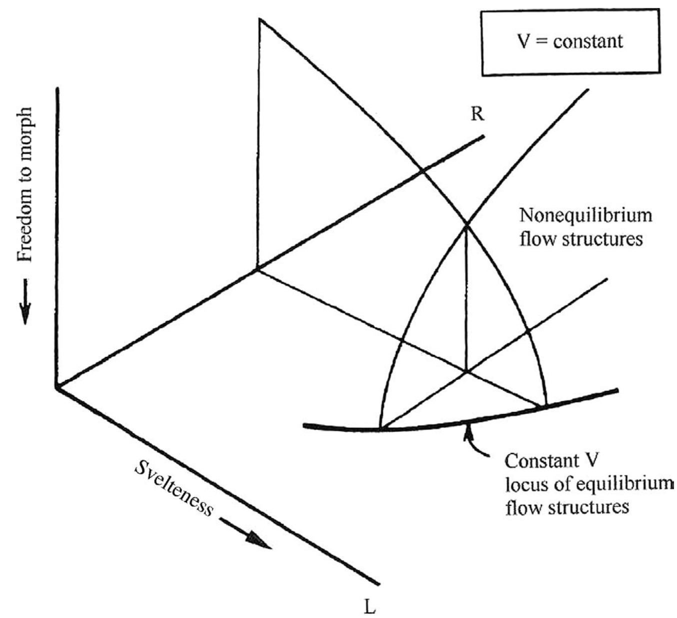

Fig. 4. The space inhabited by all the flow structures when the global internal size (V) is fixed [5].

is physics, and such is life. It is harder to drive through a densely populated urban area than through a peripheral quarter. But, we drive as well as we do because, given a population density, we generate tree-shaped flow structures. We morph our movement in time in accordance with the constructal law.

Engineered flow configurations and all the flow designs of nature climbed and continue to climb to higher performance, toward equilibrium flow structures, in their respective freedom vs. performance worlds. Freedom is good for design performance [5].

4. Survival by increasing efficiency, territory and svelteness

All flow systems (e.g., Fig. 1) have configurations that inhabit the hyperspace suggested in Fig. 3. All the constant-L flow configurations that are possible inhabit the volume visualized by the constant-V and constant-R cuts, where R stands for flow resistance (e.g., $\Delta P/\dot{m}$ or f in Figs. 1 and 2). Plotted on the R axis is the global resistance of the flow system. The abscissa accounts for the total volume occupied by the ducts (V): this is a global measure of how 'porous' or 'permeable' the flow system is. The constant-V plane that cuts through Fig. 3 is the same as the plane of Fig. 2.

The constructal law is the statement that summarizes the common observation that flow structures that survive are those that morph in one direction in time: toward configurations that make it easier for currents to flow (see the first paragraph of Section 1). This is survival by increasing global performance, or efficiency.

The constant-R cut through the configuration space shows another way of expressing the constructal law. In the limit of total freedom, the geometry will reach another equilibrium configuration, which is represented by minimum V, or maximum svelteness. Paraphrasing the original statement of the constructal law, we may describe the evolution at constant L and R as follows:

> For a system with fixed global size and global performance to persist in time (to live), it must evolve in such a way that its flow structure occupies a smaller fraction of the available space.

This is survival by increasing global compactness, or svelteness.

The constant-V alternative to Fig. 3 is shown in Fig. 4. The constructal law statement can be read off Fig. 4 in two ways. One is the original statement [1,2]: at constant V and L, the evolution is from a nonequilibrium structure to one that has a lower global resistance. If the flow geometry continues to morph freely, the structure approaches the equilibrium configuration.

The alternative is when structural changes are made such that R remains constant while V is also fixed. Then the evolution is toward a larger L, and the constructal law statement becomes:

> In order for a flow system with fixed global resistance (R) and internal size (V) to persist in time, the architecture must evolve in such a way that it covers a progressively larger territory.

The ultimate flow structure with specified global resistance (R) and internal size (V) is the largest. This is also the most svelte. A flow architecture with specified R and V has a maximum size, and this global size belongs to the equilibrium architecture. A flow structure larger than this does not exist. This formulation of the constructal law has implications in natural design, e.g., the spreading of species and river deltas without access to the sea. We return to this observation in Section 7.

Table 1
The concepts and principles of classical thermodynamics and constructal theory [5,8]

Thermodynamics	Constructal theory
State	Flow architecture (geometry, configuration, structure)
Process, removal of internal constraints	Morphing, change in flow configuration
Properties (U, S, Vol, ...)	Global objective and global constraints (R, L, V, ...)
Equilibrium state	Equilibrium flow architecture
Fundamental relation, $U(S, \text{Vol}, \ldots)$	Fundamental relation, $R(L, V, \ldots)$
Constrained equilibrium states	Nonequilibrium flow architectures
Removal of constraints	Increased freedom to morph
Energy minimum principle:	Constructal principle:
U minimum at constant S and Vol	R minimum at constant L and V
Vol minimum at constant F and T	V minimum at constant R and L
S maximum at constant U and Vol	L maximum at constant V and R

Another way to summarize the analytical formulation that we have just constructed is by recognizing the analogy between the analytical constructal law and the analytical formulation of classical thermodynamics [1]. The analogy is presented in Table 1.

5. Multiple objectives and multiple scales

Engineering design, like animal design, represents the generation of flow configuration subject to two or more objectives. The morphing of multi-objective flow systems is a new and promising direction for constructal design. We started in this direction by deducing the optimal cavernous structure of a wall made of hollow bricks [18]: see Fig. 5. The wall design has two objectives: maximal mechanical strength, and maximal thermal resistance. These two objectives clash, and from this competition emerges the optimal number and size of wall cavities. The internal structure of the wall (the number of air caverns) can be optimized so that the overall thermal resistance of the wall is maximal, while the mechanical stiffness of the wall is fixed. The maximized thermal resistance increases when the effect of natural convection in the air gaps is weaker, and when the specified wall stiffness decreases. The optimal number of air gaps is larger when the effect of natural convection is stronger, and when the specified wall stiffness is smaller. The optimal structure is such that the volume fraction occupied by air spaces decreases when the natural convection effect (the overall Rayleigh number) increases, and when the prescribed wall stiffness increases.

The example of Fig. 5 belongs to a new class of thermal design problems, in which the system architecture is derived from a combination of heat transfer and mechanical strength considerations. This class represents an extension of the constructal design method, which until recently has been used for maximizing thermofluid performance subject to size constraints. For example, the external shape and internal structure of a beam of steel-reinforced concrete can be optimized such that the beam has mechanical stiffness and resistance to thermal attack [19]. The latter means that if the beam is suddenly exposed to intense heating, it lasts a long time until its material loses its elastic strength properties and becomes thermoplastic.

Developers of convective structures (e.g., heat exchangers, cooled electronics) seek at least two objectives: minimum

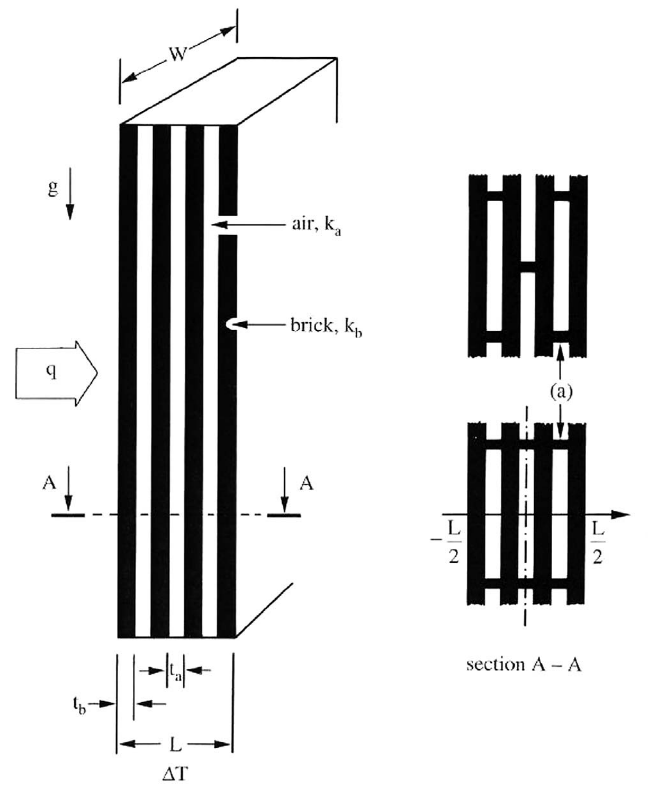

Fig. 5. Vertical insulating wall with alternating layers of solid material (brick) and air [17].

global thermal resistance and minimal pumping power. The two objectives define the two-dimensional design space shown in Fig. 6, in which the lower-left domain is preferable. Each design (flow configuration) has one curve in this plane. Curves closer to the origin can be discovered only by changing the configuration, i.e., by endowing the structure with freedom to morph (Figs. 3 and 4).

An example is shown in Fig. 7, for which the definition of the thermal resistance ($\widetilde{R}_t$) and pumping power ($\widetilde{W}$), and the construction of the figure is detailed in Ref. [14]. The curves correspond to three classes of configurations, all having laminar fully developed flow, and the same internal and external size. The solid curves in the upper-right represent two round tubes in counterflow. The dashed line is for radial counterflow between two disc-shaped sheets of fluid. The curve marked with circles

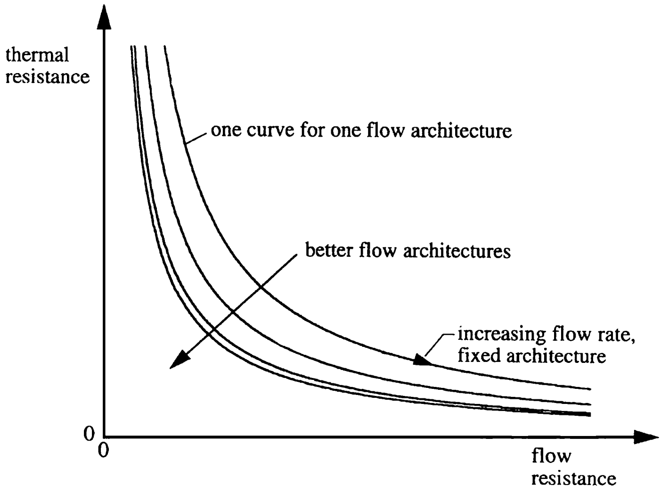

Fig. 6. Constructal route toward flow architectures with two objectives: small thermal resistance and small fluid flow resistance [13].

is the envelope (the best) of a family of curves for counterflows of two disc-shaped trees, one curve for a number (n) of levels of pairing or bifurcation. The curve marked with squares is the envelope of the family of curves obtained for tree counterflows covering a square, not a disc.

Fig. 7 is the clearest argument to date in favor of tree-shaped architectures for transport devices, because trees provide low thermal resistance and low pumping power at the same time. Current heat exchanger methodology is based on assuming one-scale flow structures such as the single-D structures shown in the two insets in Fig. 7. Tree-shaped architectures have multiple scales (D_i, L_i), which are organized hierarchically, and which can be distributed optimally (and nonuniformly) over the available territory, and their performance is orders-of-magnitude superior. This is why the tree-shaped flow architectures proposed originally for conduction [20] are now pursued in electronics cooling [4,21] and fuel cells [22–24].

The migration of the design curves from the upper-right toward the lower-left in Fig. 7 is possible because of the freedom to change the configuration. In this direction, the number of degrees of freedom increases, and the best flow structures (trees) exhibit optimized complexity. The connection between Fig. 7 and, say, Fig. 2 is this: Fig. 7 is the base plane of a three-dimensional space in which the third direction is the freedom axis, as in Fig. 3. If one of the two objectives becomes a constraint (e.g., constant thermal resistance), then the design space reduces to the cut made at $\widetilde{R}_t =$ constant in Fig. 7. In this cut, we have a freedom vs. performance diagram like Fig. 2, with $\widetilde{W}$ on the abscissa, and the freedom to change the configuration on the ordinate.

Structures with multiple scales that are distributed nonuniformly do not have to be tree-like (dendritic). An example is the augmentation of heat transfer from a wall with discrete heat sources [25–28], e.g. Fig. 8. For this we showed analytically how to optimize the spacings between heat sources, and found that near the tip of every boundary layer the best spacing is zero. There is a starting region of length x_0 along which the heat sources must be positioned flush next to each other. Even

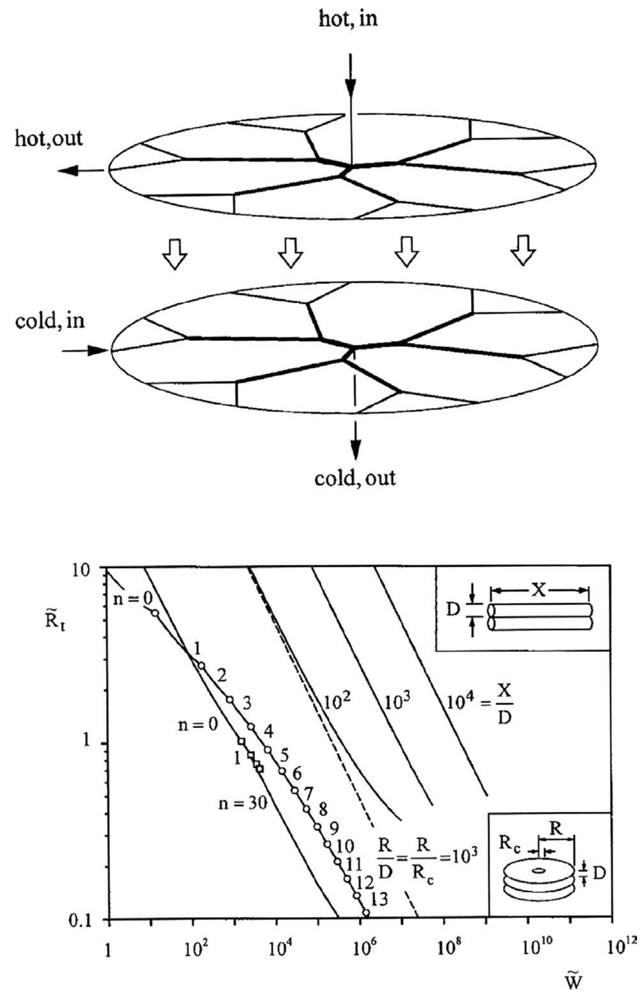

Fig. 7. Two disc-shaped tree flows in counterflow, and the design space of global thermal resistance vs. pumping power [14].

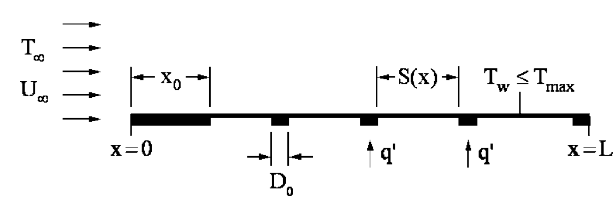

Fig. 8. Wall with multi-scale nonuniform distribution of heat sources [26].

when the heat sources themselves have a single length scale (D_0), the flow architecture has multiple length scales (x_0, S_i) that are distributed nonuniformly: x_0 and the smallest spacings are positioned near the start of the flow region. Additional structures with multiple scales that are distributed nonuniformly are reviewed in Ref. [4].

6. Cost of configuration

There is yet another dimension to be added to the design space, and it accounts for the effort made as the nonequilibrium flow structure progresses toward high global performance (equilibrium flow structure, Fig. 3). It is important to think of computational cost, and how to reduce it by devising effective

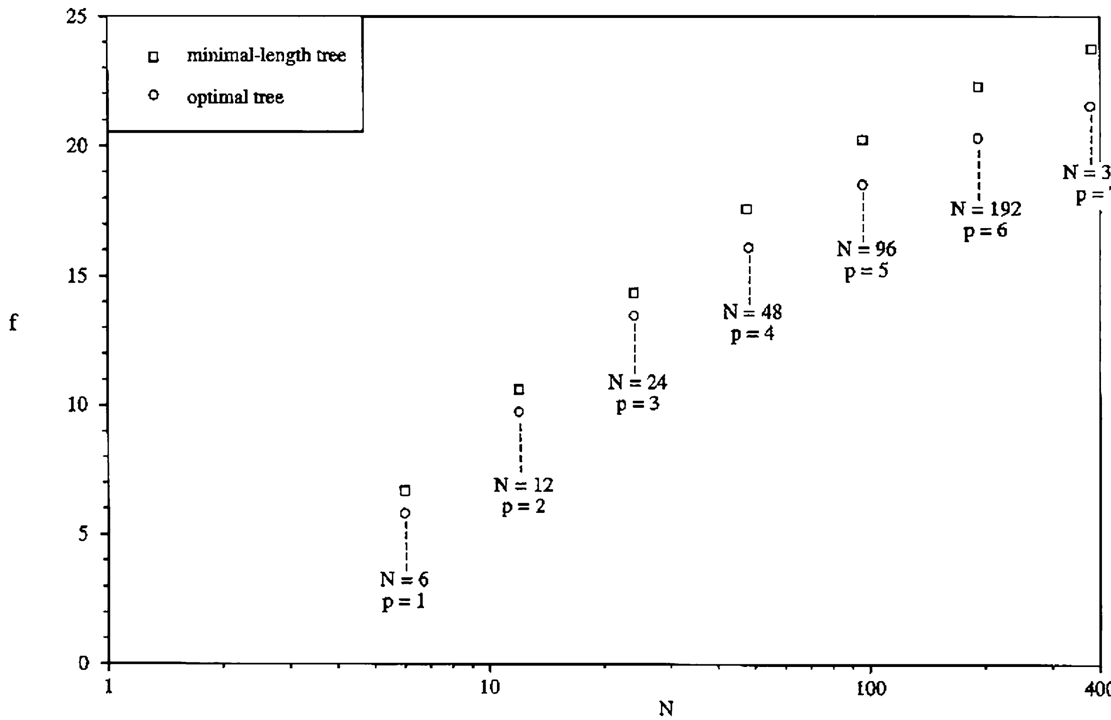

Fig. 9. The lowest global flow resistances of trees-on-disc flows constructed by using two different methods [12].

strategies for identifying paths (preferable features) that lead more directly to equilibrium (or near-equilibrium) flow structures [29].

As an example, consider the class of point-circle tree flows of Figs. 1 and 7. The best designs (lowest points) of Fig. 1 are represented by circles in Fig. 9. We had discovered in Ref. [12] that it is possible to generate very rapidly near-optimal tree structures by minimizing the length of every duct that is allocated to an area element of the disc. This is the same as optimizing the shape of the area element allocated to the duct. The performance of the minimal-length designs is indicated by squares in Fig. 9. The performance of minimal-length trees is 10–20 percent inferior to that of the equilibrium structures, but their cost is orders-of-magnitude smaller than for equilibrium structures, especially as the number of pairing levels (p) increases.

We also found that the optimal angle of bifurcation of a Y-shaped construct of three tubes with Poiseuille flow is very close to 75° [11,16,30]. This 75°angle is visible in optimal tree architectures with large p (Fig. 1), especially near the rim. This finding suggests the strategy to set all the pairing angles equal to 75°, and generate the tree-on-disc architecture even faster [29].

7. Conclusions

An important conclusion is that it is not always necessary to generate the fully optimized flow structure. It pays to travel on the axis of freedom with strategy, in order to bring the flow configuration close to the equilibrium structure, and to do it with negligible cost.

The main message of this line of investigation is condensed in Figs. 3 and 4. The original statement of the constructal law was about the maximization of flow access under global size constraints (external L, internal V). This means survival by increasing efficiency—survival of the fittest. This is the physics principle behind Darwin's observations, the principle that rules not only the animate natural flow systems, but also the inanimate natural flow systems and the engineered flow systems. The engineered systems are diverse species of "man + machine" beings. Engineering is the biology of "man + machine species".

An equivalent interpretation of the constructal law is that flow architectures with the same performance (R) and size (L) evolve toward compactness—smaller volumes dedicated to the internal ducts, i.e., larger volumes dedicated to the working volume elements, which are the interstices. This is survival based on the maximization of the use of the available space, i.e. the *maximization of svelteness*.

A third interpretation is survival by spreading: growth as the mechanism for being able to persist in time. The limit to growth is set by the specified constraints, in this case the fixed global flow resistance R and the global internal size V. A given flow structure or living species (river delta, animal population, Roman empire) will spread over a certain, maximal territory, where it achieves maximal svelteness. Now we know why this should be so.

References

[1] A. Bejan, Advanced Engineering Thermodynamics, second ed., Wiley, New York, 1997.
[2] A. Bejan, Shape and Structure, from Engineering to Nature, Cambridge University Press, Cambridge, 2000.
[3] H. Poirier, Une théorie explique l'intelligence de la nature, Science & Vie 1034 (November 2003) 44–63.

[4] A. Bejan, I. Dincer, S. Lorente, A.F. Miguel, A.H. Reis, Porous and Complex Flow Structures in Modern Technologies, Springer, New York, 2004.
[5] A. Bejan, S. Lorente, The constructal law and the thermodynamics of flow systems with configuration, Int. J. Heat Mass Transfer 47 (2004) 3203–3214.
[6] R.N. Rosa, A.H. Reis, A.F. Miguel (Eds.), Bejan's Constructal Theory of Shape and Structure, Évora Geophysics Center, University of Évora, Portugal, 2004.
[7] A. Bejan, S. Lorente, A course on flow-system configuration and multi-scale design, in: ASME Paper IMECE2004-59203, International Mechanical Engineering Congress and Exposition, Anaheim, California, 14–19 November 2004.
[8] A. Bejan, S. Lorente, La Loi Constructale, L'Harmattan, Paris, 2005.
[9] W. Wechsatol, S. Lorente, A. Bejan, Tree-shaped insulated designs for the uniform distribution of hot water over an area, Int. J. Heat Mass Transfer 44 (2001) 3111–3123.
[10] W. Wechsatol, S. Lorente, A. Bejan, Development of tree-shaped flows by adding new users to existing networks of hot water pipes, Int. J. Heat Mass Transfer 45 (2002) 723–733.
[11] W. Wechsatol, S. Lorente, A. Bejan, Optimal tree-shaped networks for fluid flow in a disc-shaped body, Int. J. Heat Mass Transfer 45 (25) (2002) 4911–4924.
[12] S. Lorente, W. Wechsatol, A. Bejan, Tree-shaped flow structures designed by minimizing path lengths, Int. J. Heat Mass Transfer 42 (2002) 3299–3312.
[13] W. Wechsatol, S. Lorente, A. Bejan, Dendritic convection on a disc, Int. J. Heat Mass Transfer 46 (2003) 4381–4391.
[14] A.K. da Silva, S. Lorente, A. Bejan, Constructal multi-scale tree-shaped heat exchangers, J. Appl. Phys. 96 (3) (2004) 1709–1718.
[15] W. Wechsatol, S. Lorente, A. Bejan, Tree-shaped networks with loops, Int. J. Heat Mass Transfer 48 (2005) 573–583.
[16] A. Bejan, L.A.O. Rocha, S. Lorente, Thermodynamic optimization of geometry: T- and Y-shaped constructs of fluid streams, Int. J. Thermal Sci. 39 (2000) 949–960.
[17] L.A.O. Rocha, S. Lorente, A. Bejan, Constructal design for cooling a disc-shaped area by conduction, Int. J. Heat Mass Transfer 45 (2002) 1643–1652.
[18] S. Lorente, A. Bejan, Combined 'flow and strength' geometric optimization: internal structure in a vertical insulating wall with air cavities and prescribed strength, Int. J. Heat Mass Transfer 45 (2003) 3313–3320.
[19] L. Gosselin, S. Lorente, A. Bejan, Combined 'heat flow and strength' optimization of geometry: mechanical structures most resistant to thermal attack, Int. J. Heat Mass Transfer 47 (2004) 3477–3489.
[20] A. Bejan, Constructal-theory network of conducting paths for cooling a heat generating volume, Int. J. Heat Mass Transfer 40 (1997) 799–816, (issue published on 1 November 1996).
[21] D.V. Pence, Reduced pumping power and wall temperature in microchannel heat sinks with fractal-like branching channel networks, Microscale Thermophys. Engrg. 6 (2002) 319–330.
[22] J.V.C. Vargas, A. Bejan, Thermodynamic optimization of internal structure in a fuel cell, Int. J. Energy Res. 28 (2004) 319–339.
[23] S.M. Senn, D. Poulikakos, Tree network channels as fluid distributors constructing double-staircase polymer electrolyte fuel cells, J. Appl. Phys. 96 (1) (2004) 842–852.
[24] J.V.C. Vargas, J.C. Ordonez, A. Bejan, Constructal flow structure for a PEM fuel cell, Int. J. Heat Mass Transfer 47 (2004) 4177–4193.
[25] A.K. da Silva, S. Lorente, A. Bejan, Optimal distribution of discrete heat sources on a wall with natural convection, Int. J. Heat Mass Transfer 47 (2004) 203–214.
[26] A.K. da Silva, S. Lorente, A. Bejan, Optimal distribution of discrete heat sources on a plate with laminar forced convection, Int. J. Heat Mass Transfer 47 (2004) 2139–2148.
[27] A.K. da Silva, S. Lorente, A. Bejan, Maximal heat transfer density in vertical morphing channels with natural convection, Numer. Heat Transfer, Part A 35 (2004) 135–152.
[28] A.K. da Silva, S. Lorente, A. Bejan, Constructal multi-scale structures with asymmetric heat sources of finite thickness, Int. J. Heat Mass Transfer 48 (2005) 2662–2672.
[29] W. Wechsatol, S. Lorente, A. Bejan, Tree-shaped flow architectures: Strategies for increasing optimization speed and accuracy, Numer. Heat Transfer, Part A 36 (2005), in press.
[30] L. Gosselin, A. Bejan, Tree networks for minimal pumping power, Int. J. Thermal Sci. 44 (2005) 53–63.